Intellectual Property Rights and National R&D Subsidy Policies in a Two-Country Schumpeterian Framework.

Piotr Stryszowski
Tilburg University

April 10, 2005

Abstract

I present a two-country Schumpeterian growth model without scale effect, where both countries converge to parallel growth paths because of technological transfer. Two instruments are used by the lagging country to improve its position: R&D subsidies and improvement of patent protection. Because of additional effect on the labor market, the intellectual property protection tends to have more impact on country's relative position in the world's productivity rank than the direct subsidies to research. (JEL O1, O3)

1 *Introduction*

On of the main concerns of the modern growth theory is the issue of the long - run effectiveness of R&D policy. The main question discussed in the literature is whether a policymaker can significantly and permanently influence the rate of economic growth by the appropriate R&D subsidy policy. The discussion was initiated by papers of Grossman and Helpman (1991) and Aghion and Howitt (1992) who stressed the importance of research and technology in final output creation. This claim about the efficiency of R&D subsidies on economic growth was criticized by Jones (1995) who pointed that the assumptions made in these models on the cumulative nature of R&D together with input factor growth should lead to the explosion of growth that is not observed in the reality. This critique has been incorporated into the theory by Young (1998) and

Segerstrom (1998), who extended the initial models by adding the horizontal expansion to the economy and introducing the increasing difficulty of research.

The last piece of research on the effectiveness of R&D policies on economic growth was presented by Howitt (1999). He created a theoretical Schumpeterian model of growth that is consistent with most of the empirical phenomena pointed by Jones. Howitt's model captured all the key elements that have been introduced in the earlier studies - the factors of production are growing and the economy develops in two ways - horizontally (variety expanding) and vertically (quality improving). One of Howitt's conclusion is that an increase in R&D subsidy leads to a faster vertical improvement and hence boosts the economic growth. The most important messages is that the policy can possibly affect positively the rate of growth. Following this message a planner can use the subsidies to R&D in aiming at the permanent increase of the long run growth rate of given country. Howitt (2000) continued this way of analysis investigating the reasons of potential differences in productivity between the countries. His claim was that apart form various R&D subsidies, these differences could be explained by differences in capital stocks.

The present paper contributes to the literature by analyzing the effects of different patent regimes and R&D subsidies on countries' positions in productivity rank. It also reformulates the original model of Howitt (1999) by adding the microfoundations as presented by Grossman and Helpman (1991).

This study has two main purposes:

- to study the effect of two independent research policies (patent policy and subsidies to R&D) on the economic performance of a lagging country.
- to explain the relative position of a country in the world productivity rank by these terms.

Similarly to Howitt I create Schumpeterian model of growth with both - horizontal and vertical expansions. One country reports higher aggregate productivity. The lagging country relies on copying of the discoveries made by the leader in the past. The process of copying requires some resources to be spent. The costliness of copying is an observed phenomenon showed in the literature by Coe and Helpman (1995) or by Benhabib and Spiegel (2002). Costliness implies that once a country wants to create a new generation of a products within given range it must pay the cost proportional to the cost that has been paid in order to make this discovery. This is captured by the *index of copying difficulty.* This index depends positively on the technology of the sectors in the leading country and negatively on the technology of sector in the following country. Hence the more given sector is lagging the easier is to make one step towards the frontier.

Subsidies and patent policy translate into economic development. The subsidies to R&D decrease the costs of research, hence higher subsidy rate increases investments in vertical R&D (by diminishing returns horizontal is constant) and thus moves country's position to the point where the decrease in R&D costs gets offset by increased difficulty of copying. The property rights increase the vertical R&D through two channels - by increasing the value of potential discovery and by changing the structure of the market.

A result is that a change in the R&D policy of the lagging country pushes this country up, towards the frontier so that after given period of transition it ends up in a new steady state, closer to the productivity leader. Similarly if the follower improves the patents this translates into a new, higher position in the productivity rank. Clearly if the leading country improves the patents or increases subsidies and notices faster growth, this leads to a deterioration of the position of the lagging country.

This paper is structured as follows. The next section sketches the theoretical model. Section three presents the dynamics of the model. The last section discusses the main results and concludes.

2 *The Model*

There are two structurally identical countries (identical factor supplies at each point at time). As in Howitt (1999) the only possible international internal interaction is the technological transfer. The productivity for the frontier country grows at the exogenous rate g^F.

2.1 Industry structure

There's a continuum of industries $i \in [0, B_t)$. There are two basic activities in the economy: production and R&D. Firms engage in R&D to:

- discover higher quality products in all industries throughout time (vertical R&D),

- open new industries (horizontal R&D).

2.2 *Individuals*

The number of individuals (L_t) - same for both economies - grows over time according to an exogenous parameter g_L; hence,

$$L_t = e^{g_L t}$$

The individuals (dynastic families) maximize

$$U = \int_0^\infty e^{g_L t} e^{-\rho t} \ln u(c_t) dt$$

where $u(c_t)$ denotes the individual utility form consumption at t and ρ is the rate of time preference. The individual utility at t is equal to:

$$\ln u(c_t) = \int_0^{B_t} \ln c_t(i) di$$

where $c_t(i)$ denotes the consumption of products from sector i. As shown in the appendix consumption expenditures are spread equally among the existing sectors so that:

$$c_t(i) = \frac{E_t}{p_t(i) B_t}$$

hence consumers choose the cheapest products from each industry. The constant expenditure path is optimal if and only if the interest rate is equal to:

$$r = (\rho - g_L) \tag{1}$$

2.3 *Production*

The sector of *production* consists of continuum of industries (denoted by i) and uses labor as the only input. One worker at time t can produce $A_t(i)$ goods in industry i. $A_t(i)$ changes as a result of technological progress. Particularly assume, that every new discovery increases $A_t(i)$ by an exogenous factor $\lambda > 1$.

A company that possesses the current state-of-the-art technology sets its price to make the consumers weakly preferring its products over its competitors' in given industry branch. In case when there are no previous monopolists (i.e. a sector was newly created), there is a competitive fringe that can enter at any time and produce using a technology that is worse by λ than the technology used by the incumbent. Since competition drives the prices down to the marginal costs and consumers purchase only the cheapest good, it is optimal for the current monopolist to set the price equal to the marginal cost of the competitor $w_t\lambda/A_t(i)$. I normalize the wages to be equal unity, so that in an unleveled sector the prices are equal to $\lambda/A_t(i)$. Thus the monopolist earns a stream of profits equal to:

$$\pi_t = \left(1 - \frac{1}{\lambda}\right)\frac{E_t}{B_t} \tag{2}$$

Let $V_t(i)$ be the value of firm (monopolist) in i. This value is the stream of profits discounted by the subjective rate of time - preference, rate of population growth and adjusted for the probability of losing the monopolistic power. Thus:

$$V_t(i) = \int_t^\infty \pi_\tau \exp - \left[\int_t^\tau (\rho + g_L + \phi_s(i))\, ds\right] d\tau \tag{3}$$

where $\phi_s(i)$ is the instantaneous probability of losing the monopolistic power at s. Note that when there's no monopolist in given sector i (i.e. current monopolist lost its position before the succeeding discovery) Bertrand competition drives the prices to the marginal costs of production $p_t(i) = w_t/A_t(i) = 1/A_t(i)$.

2.4 *R&D*

The *sector of R&D* also uses labor as the only input. Research can be *vertical* (productivity improving) or *horizontal* (variety expanding). Let β be the R&D subsidy rate.

Furthermore suppose that the patents in given country are imperfect. This imperfection of the quality of protection of intellectual property implies that at each point of time there's a probability σ that the Patents are abundant. If a company looses its patent in given sector before the succeeding discovery occurs, it means that given sector becomes leveled and Bertrand competition drives the prices to the marginal costs of production $p_t(i) = 1/A_t(i)$.

2.4.1 *Vertical R&D*

Every *vertical* innovation in industry i increases the labor output by an exogenous factor $\lambda > 1$. A successful company benefits a stream of monopolistic profits in given industry until it is replaced by next discovery or till the patent stops working and given sector becomes leveled.

The process of copying becomes more difficult the closer to the frontier given industry is. Particularly let $x_t^C = A_t(i)/A_t^F(i)$ be the difficulty parameter - that denotes the distance of technological level in given industry $(A_t(i))$ to the frontier $(A_t^F(i))$. Recall that $\dot{A}_t^F(i)/A_t^F(i) = g^F$.

Labor is the only input in vertical R&D and free entry is assumed. Any R&D firm that hires $n_t(i)$ units of labor in industry i at t is successful in discovery of the next higher - quality product with probability $\left[\phi_v n_t(i)/x^C(i)\right]$ where $\phi_v > 0$ is the productivity parameter of vertical innovations.

Denote by $V_t(i)$ the value of vertical innovation equal in sector i. By no-arbitrage condition the marginal revenue of vertical research $([V_t(i)\phi_v])$ must equal to the marginal costs of research. Introduction of the R&D subsidy rate β implies the marginal costs of $(1-\beta)$, hence:

$$V_t(i)\phi_v/x^C(i) = (1-\beta)$$

2.4.2 *Horizontal R&D*

Horizontal R&D also uses labor in research, but due to some specific skill requirements (as in Howitt, 1999) it exhibits the decreasing returns to scale properties. The success in horizontal R&D results in establishing of new industry lab in the sector of manufacturing and depends positively on volume of labor employed (with diminishing marginal effect). New monopolist enjoys a profit stream facing a competitive fringe until it is displaced by next discovery. Assume that the technological level of a newly established industry is randomly drawn from technological levels of existing products. Similar process of creation of new industries is observed in the frontier country, and therefore in order to control for differences in productivity, the process of horizontal expansion is adjusted by the distance to the frontier $1/x_t^C$.

Denote the rate of new products innovation as:

$$\begin{aligned} \dot{B}_t &= N_{ht}^{\psi}, L_t^{1-\psi}/x_t^C \\ \dot{B}_t &= h_t^{\psi} L_t/x_t^C \end{aligned}$$

where:

- h_t is the fraction of labor employed in horizontal research $h_t \equiv N_{ht}/L_t$,
- ψ is the productivity parameter of horizontal research, $\psi \in (0.1)$.

3 *Dynamics of the model*

Let:

- $L_t/B_t \equiv l_t$ denote the number of labor units per sector in F (L),
- $N_{ht}/L_t \equiv h_t$ be the share of population employed in horizontal research,
- $\int_0^{B_t^F} n_t(i)/B_t \equiv n_t$ be the average intensity of vertical R&D in F (L),
- γ_t fraction of sectors that are unleveled (i.e. sectors with a monopoly with the patent pending).

The non-arbitrage condition between the horizontal / vertical research implies:

$$\begin{aligned} V_t \psi h_t^{\psi-1}/x_t^C &= V_t \phi_v / x_t^C \\ \psi h_t^{\psi-1} &= \phi_v \\ h_t &= \left(\frac{\phi_v}{\psi}\right)^{\frac{1}{\psi-1}} \end{aligned}$$

Hence the fraction of population devoted to horizontal R&D (h) is constant over time. Recall that $L_t/B_t \equiv l_t$ denotes the number of labor units per sector. Thus:

$$\begin{aligned} \frac{\partial}{\partial t}\ln l_t^L &= \left[\frac{\partial}{\partial t}\left(L_t/B_t\right)\right] / \left[L_t/B_t\right] \\ \frac{\partial}{\partial t}\ln l_t^L &= \left(\dot{L}_t/L_t\right) - \left(\dot{B}_t/B_t\right) \\ \frac{\partial}{\partial t}\ln l_t^L &= g_L - h_t^{\psi} L_t/B_t \end{aligned} \tag{4}$$

since h_t is constant in the frontier country (4) implies that l_t^L converges to:

$$l^L = g_L \left(\frac{\psi}{\phi_v}\right)^{\frac{\psi}{\psi-1}} \tag{5}$$

Assume that this convergence occurred, hence (5) holds. This means that in the lagging country the values of l and h are constant over time. In other words the fraction of population devoted to horizontal R&D and number of people per sector are constant.

Now I turn to the labor market. Every agent can choose between being employed in production or in a research company. Thus, total population consists of people employed in production, horizontal and vertical research. Therefore the labor market equation can be presented as follows:

$$\begin{aligned}
\underbrace{\int_0^{B_t} \frac{c_t(i)}{A_t(i)} di}_{\text{production}} + \underbrace{\int_0^{B_t} n_t(i) di}_{\text{vertical research}} + \underbrace{N_{ht}}_{\text{horizontal research}} &= L_t \\
\int_0^{B_t} \frac{E_t}{p_t(i) B_t} \frac{1}{A_t(i)} di + \int_0^{B_t} n_t(i) di + N_{ht} &= L_t
\end{aligned}$$

recall that $p_t(i) = \lambda / A_t(i)$ if sector is unleveled and $p_t(i) = 1/A_t(i)$ if leveled. Thus:

$$\begin{aligned}
\int_0^{B_t} \frac{E_t}{B_t} \frac{1}{(\gamma_t (\lambda - 1) + 1)} di + \int_0^{B_t} n_t(i) di + N_{ht} &= L_t \\
\frac{1}{(\gamma_t (\lambda - 1) + 1)} E_t + \int_0^{B_t} n_t(i) di + N_{ht} &= L_t
\end{aligned}$$

By symmetry between the sectors $\int_0^{B_t} n_t(i) = n_t B_t$ hence:

$$\begin{aligned}
\frac{1}{(\gamma_t (\lambda - 1) + 1)} E_t + n_t B_t + N_{ht} &= L_t \\
\frac{1}{(\gamma_t (\lambda - 1) + 1)} \frac{E_t}{L_t} + n_t / l + h &= 1 \\
(1 - n_t / l - h) (\gamma_t (\lambda - 1) + 1) &= \frac{E_t}{L_t}
\end{aligned}$$

Hence, the monopolistic profits as described by (2) are:

$$\pi_t = \left(1 - \frac{1}{\lambda}\right) \left(1 - \frac{n_t}{l} - h\right) (\gamma_t (\lambda - 1) + 1) l$$

Note, that γ denotes the fraction of sectors that are unleveled. The only case when a sector becomes leveled is when the patent expires before the new discovery occurs. Hence

$$\gamma_t = \frac{n_t \phi_v / x_t^C}{n_t \phi_v / x_t^C + \sigma} \tag{6}$$

According to (3) the value of the new discovery is:

$$V_t(i) = \int_t^{\infty} \pi_\tau \exp - \left[\int_t^{\tau} (\rho - g_L + \phi_s(i)) ds \right] d\tau \tag{7}$$

Differentiating (7) with respect to time yields:

$$\left(\pi_t + \dot{V}_t\right) / V_t - n \phi_v / x_t^C - \sigma = \rho - g_L \tag{8}$$

So that for a holder of the stock of a monopolist every moment of time brings profits of π_t and appreciation of stock value of $\dot{V}_t$. In case of new discovery or loss of patent the stockholder suffers from a loss of $V_{t.}$ This happens if a new discovery occurs (with probability $n\phi_v/x_t^C$) or if given sector becomes leveled (with probability σ). This must be equal to the market interest rate that by (1) equals to the subjective discount factor (ρ) minus the rate of population growth (g_L).

By non-arbitrage the value of a discovery must be equal to the opportunity cost of research (i.e. to the unity wage) adjusted for the probability of achieving given discovery (ϕ_v/x_t^C) plus the marginal subsidy rate (β):

$$\begin{aligned} 1 &= V_t\phi_v/x_t^C + \beta \\ V_t &= (1-\beta)\,x_t^C/\phi_v \end{aligned} \tag{9}$$

Note that the growth of value of a monopolist is therefore:

$$\begin{aligned} \dot{V}_t/V_t &= \dot{x}_t^C/x_t^C \\ \dot{V}_t/V_t &= \dot{A}_t/A_t - \dot{A}_t^F/A_t^F \\ \dot{V}_t/V_t &= n_t \ln\lambda - g_F \end{aligned} \tag{10}$$

So the growth of the value of a monopolist equals to the rate technological progress of given country minus the growth of the technological frontier.

Plugging (9) and (10) into equation (8) gives:

$$\left(\frac{1}{x_t^C}\frac{\pi_t\phi_v}{(1-\beta)} + n_t\ln\lambda - g_F\right) = \rho - g_L + \sigma + \frac{1}{x_t^C}n_t\phi_v$$

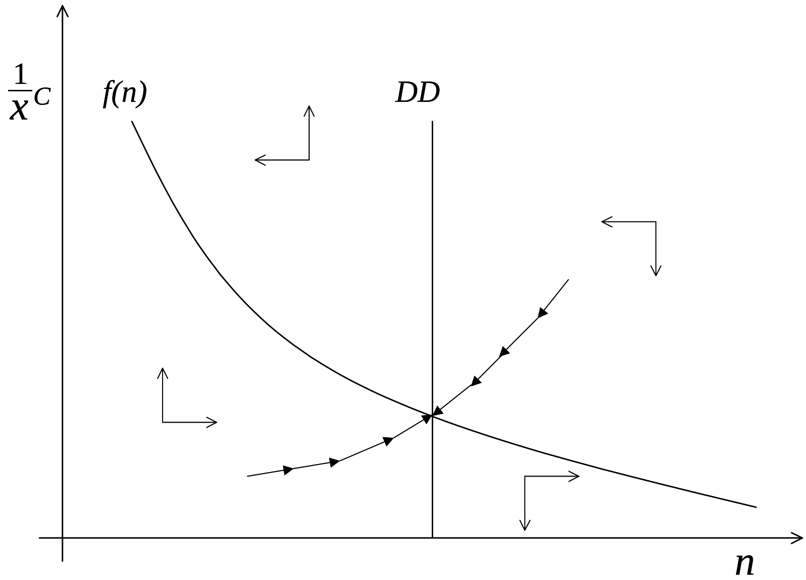
I analyze the system in the $1/x^c, n$ plane on figure one. Recall that the term $1/x^c$ represents the relative position of given country in the world productivity rank (i.e. its distance to the frontier) and n is the per-sector research intensity.

Referring to (10) the growth rate of the distance to the frontier ($1/x^c$) equals to:

$$\left(\frac{\partial}{\partial t}\frac{1}{x_t^C}\right)x^c = g_F - n_t\ln\lambda$$

The distance to the frontier is constant if $g_F = n^*\ln\lambda$, i.e. when the research the intensity of vertical research equals to:

$$n^* = \frac{g_F}{\ln\lambda}$$

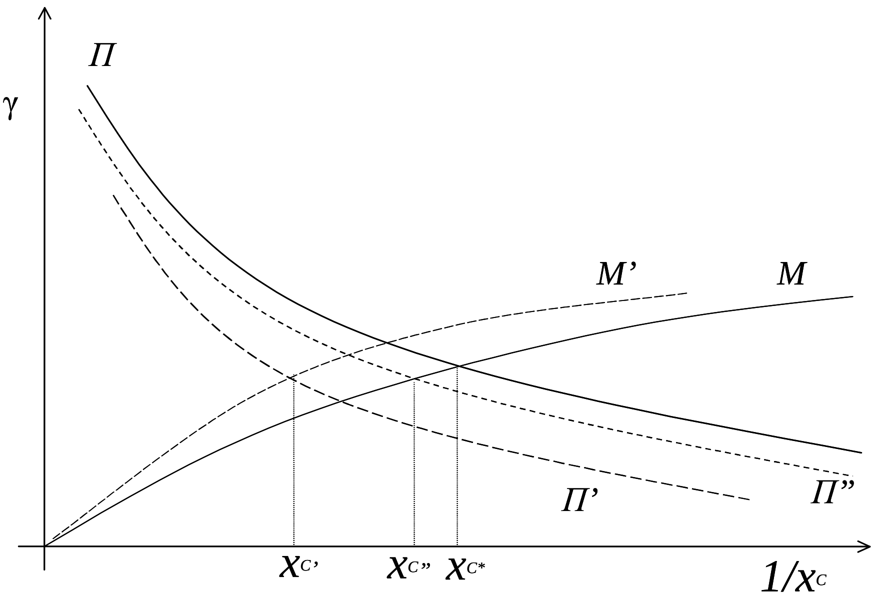


On figure one this is by the vertical DD line. All points on the left indicate less R&D intensity that is required to sustain constant distance to the frontier, and thus growth of $1/x^c$. All points on the right of DD imply higher R&D than the frontier's R&D and hence convergence towards the frontier.

In order to identify the second equation I use equation (8) study the case when n is constant. As shown in the appendix transforming (8) yields function $1/x_t^C = f(n)$, where $f(0) = \infty$, $f' < 0$. For all points above (below) this line there is too much (too little) vertical R&D in order to balance for the non - arbitrage equation to hold.
As shown on figure one there's only one stable equilibrium point. Divergence towards $n \to \infty$ is impossible as n is bounded form above (with the volume of population). Divergence towards $n \to 0$ implies that in the limit country stagnates completely, hence it violates the transversality condition. The long run steady state is indicated by the intersection of the curves DD and $f(n)$. There are two stable saddle patches that imply that in the long run given economy converges to this equilibrium point.

4 *Discussion*

The long run steady state presented in the previous section determines the country's steady state relative productivity and its intensity of vertical research. Given that the steady state in stable in the long run I analyze what are the consequences of country's research policy (R&D subsidies and patent protection) on the distance to the technological



frontier.

Transforming equation (8) yields:

$$\left(\frac{1}{x^C}\tilde{\pi}\left(\gamma_t\left(\lambda-1\right)+1\right)\phi_v+n_t\ln\lambda-g_F\right)=\rho-g_L+\sigma+\frac{1}{x_t^C}n_t\phi_v$$

where

$$\tilde{\pi}=\left(1-\frac{1}{\lambda}\right)\left(1-\frac{n}{l}-h\right)l/\left(1-\beta\right)$$

hence:

$$\gamma_t=x^C\frac{\rho-g_L+\sigma}{\phi_v\tilde{\pi}\left(\lambda-1\right)}+\frac{n-\tilde{\pi}}{\tilde{\pi}\left(\lambda-1\right)} \tag{11}$$

On the $1/x^C,\gamma$ plane (figure two) equation (11) is a downward sloping curve denoted by Π. The steady state point can be determined with the equation (6) that defines the shares of monopolies on the market. This equation is denoted on figure two with the upward sloping M curve. The intersection of the curves M and Π determines the equilibrium point. with the corresponding distance to the frontier $1/x^{C*}$.
An increase of the subsidy rate (β) increases $\tilde{\pi}$ hence the Π curve shifts down to Π". Cheaper research becomes more attractive thus the research intensity rows and the distance to the frontier shrinks. The economy converges to the new steady state point $1/x^{c"}$.

An increase of the quality of patents has also positive effect on the reduction of the distance to the technological frontier. However, the transmission mechanism occurs through two channels. Better patents increase the value of new discovery (hence Π shifts to the left). Besides better patents imply more monopolies on the market, thus the employment structure changes so that more people gets employed in the research companies (M shifts to the left). The new equilibrium point $(1/x^{c\prime})$ is determined by the intersection of the M' and Π' curves.

5 *Concluding Remarks*

To reiterate, this paper presents a model that stresses the importance on intellectual property rights on countries' relative position in the world's productivity rank. By increasing the R&D subsidy rate or by improving the patent protection a planner can shift the aggregate level of productivity of her country towards the technological frontier.

According to the theoretical prediction, intellectual property rights seem to have more effect on the growth rate than the direct subsidies to R&D. The subsidies affect the R&D incentives only through diminishing the cost of research. A more promising instrument is the patent policy - as it stimulates the creation of new ideas through two channels. Firstly, it increases the profitability of R&D by extending the period of profit gains. Secondly, by the increase of number of monopolies on the market it changes the composition of the labor market by shifting people towards R&D. Note, that all these results have been presented using a standard schumpeterian model of creating destruction without scale effect.

References

[1] Aghion, P. and Howitt, P., (1992), "A Model of Growth through Creative Destruction", *Econometrica*, 60

[2] Benhabib, J. and Spiegel, M. (2002), "Human Capital and Technology Diffusion" FRSBF Working Paper #2003-02

[3] Coe, D. T. and Helpman, E., (1995), “International R&D Spillovers,” *European Economic Review* 39

[4] Grossman, G., Helpman, E., (1991), ”Quality Ladders in the Theory of Growth”, *Review of Economic Studies*, vol. 58, pp. 43-61

[5] Howitt, P., (1999), "Steady Endogenous Growth with Population and R&D Inputs Growing", *Journal of Political Economy*, 107 / 4

[6] Howitt, Peter, Endogenous Growth and Cross-Country Income Differences, *American Economic Review* 90 (September 2000): 829-46

[7] Jones, Ch., (1995) "R&D-Based Models of Economic Growth" *Journal of Political Economy* 103

[8] Keely,L.C., (2000) "Using patents in growth models," Working papers 30, Wisconsin Madison - Social Systems.

[9] Peretto, P. and Smulders, J. A. (2002) ""Technological Distance, Growth and Scale Effects", *Economic Journal* 112

[10] Segerstrom, P., (1998), "Endogenous Growth Withot Scale Effects", *American Economic Review* 88

[11] Young, A. (1998) "Growth Without Scale Effects" *Journal of Political Economy* 106

Demand function

$$\begin{aligned} \underset{c_t(i)}{max} \ln u(c_t) &= \int_0^{B_t} \ln c_t(i) di \\ \text{s.t.} \int_0^{B_t} p_t(i) c_t(i) di &= E_t \end{aligned}$$

Let y be the new state variable, such that: $y(0) = 0$, $y(B_t) = E_t$, $\dot{y} = p_t(i)c_t(i)$. Then the maximization problem becomes:

$$\begin{aligned} \underset{c_t(i)}{max} \ln u(c_t) &= \int_0^{B_t} \ln c_t(i) di \\ \text{s.t.} \dot{y} &= p_t(i) c_t(i) \end{aligned}$$

$H = \ln c_t(i) + \mu_t(i) p_t(i) c_t(i)$
$H_y = 0 = -\dot{\mu}$ hence μ is constant and does not depend on i.
$H_c = 1/c_t(i) + \mu_t(i) p_t(i) = 0 \Longrightarrow c_t(i) = -1/\left(\mu p_t(i)\right)$

from (1) $\int_0^{B_t} -1/\mu di = E_t \Longrightarrow -1/\mu = E_t/B_t$
hence:

$$c_t(i) = \frac{E_t}{p_t(i)B_t}$$

$$\begin{aligned} \max_{E_t} U \quad \text{where } U &= \int_0^\infty e^{-g_L t} e^{-\rho t} \ln u(c_t) dt \\ \text{s.t. } \int_0^\infty e^{-R(t)} E_t dt &= A_0 \text{ (initial assets)} \end{aligned}$$

where $\dot{R}(t)$ is the instantaneous interest rate
$\max_{E_t} \int_0^\infty e^{-g_L t} e^{-\rho t} \ln E_t \left[\int_0^{B_t} 1/p_t(i)B_t di\right] dt$
$\max_{E_t} \{ \underbrace{\int_0^\infty e^{-g_L t} e^{-\rho t} \ln E_t dt}_{\text{only this term depends on } E_t} + \int_0^\infty (...) dt \}$
$H = e^{-g_L t} e^{-\rho t} \ln E_t + \mu(t) e^{-R(t)} E_t$
$H_y = \dot{\mu} = 0$
$H_E = e^{-g_L t} e^{-\rho t}/E_t + \mu(t) e^{-R(t)} = 0 \Longrightarrow E_t = e^{R(t)-(g_L+\rho)t}/\mu(t)$

$$\frac{\dot{E}_t}{E_t} = \dot{R}(t) - (g_L + \rho)$$

Function $f(n)$.
From (8)

$$\left(\frac{1}{x_t^C} \tilde{\pi}_t \left(\gamma_t (\lambda - 1) + 1\right) \phi_v + n_t \ln \lambda - g_F\right) = \rho - g_L + \sigma + \frac{1}{x_t^C} n_t \phi_v$$

where

$$\begin{aligned} \tilde{\pi}_t &= \left(1 - \frac{1}{\lambda}\right) \left(1 - \frac{n_t}{l} - h\right) l / (1 - \beta) \\ \frac{\partial}{\partial n} \tilde{\pi}_t &< 0 \end{aligned}$$

Let:

$$f_1(n_t) \equiv 2\phi_v n_t \left(\lambda \tilde{\pi}_t + n_t\right), \; \frac{\partial}{\partial n_t} \alpha_0 > 0$$

Then

$$\frac{1}{x_t^C} = f(n_t) = f_0(n_t)/f_1(n_t)$$

where $f_1'(n_t) < 0$.